

الف) مسائل اول

Quiz

A) Prove that $\frac{d}{dz} (z\bar{z})$ doesn't exist anywhere

B) Show that $\frac{d}{dz} (\ln(z)) = \frac{1}{z}$

Complex integration

$$\int_a^b f(z) dz = \int_c f(z) dz \rightarrow \text{تكامل على مسار غير منطظم}$$

$$\int_c f(z) dz \rightarrow \text{تكامل على مسار منطظم}$$

$$\boxed{1} \quad \int_c f(z) dz = ?$$

$$f(z) = u + iv \quad ; \quad z = x + iy$$

$$dz = dx + idy$$

$$dz = dx + idy$$

$$\therefore \int_c f(z) dz = \int (u + iv) (dx + idy)$$

$$= \int (u dx - v dy) + i(v dx + u dy)$$

17 Sect

نجهی \Rightarrow ①
او در برای میسر است dy, y_0, dx, x_0 که نجهی \Rightarrow ①

① Circle $(x - x_0)^2 + (y - y_0)^2 = a^2$

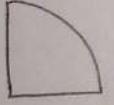
$$x = x_0 + a \cos t \quad ; \quad y = y_0 + a \sin t$$

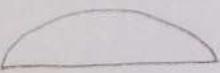
$$dx = -a \sin t dt \quad ; \quad dy = a \cos t dt$$

② قطع ناقص $\frac{(x - x_0)^2}{a^2} + \frac{(y - y_0)^2}{b^2} = 1$

$$x = x_0 + a \cos t \quad y = y_0 + b \sin t$$

$$dx = -a \sin t dt \quad dy = b \cos t dt$$


 $0 \leq t \leq \frac{\pi}{2}$


 $0 \leq t \leq \pi$


 $0 \leq t \leq 2\pi$ \Leftarrow المدورة

③ قطع زوایا $\frac{(x - x_0)^2}{a^2} - \frac{(y - y_0)^2}{b^2} = 1$

$$x = x_0 + a \cosh t$$

$$y = y_0 + b \sinh t$$

$$dx = a \sinh t dt$$

$$dy = b \cosh t dt$$

② Sec 7

④ 

$$y = a x^2$$

الخط المستقيم ③

$$x = t$$

$$y = a t^2$$

$$dx = dt$$

$$dy = 2at dt$$

⑤

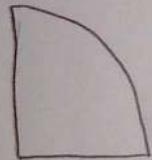
$$|z - z_0| = a$$

$$x = x_0 + a \cos \theta$$

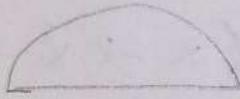
$$y = y_0 + a \sin \theta$$

$$dx = -a \sin \theta d\theta$$

$$dy = a \cos \theta d\theta$$



$$0 \leq \theta \leq \frac{\pi}{2}$$



$$0 \leq \theta \leq \pi$$



$$0 \leq \theta \leq 2\pi$$

□ Evaluate $\int f(z) dz$ From 0+ic to 2+5i

where $f(z) = (3x+y) + i(x-2y)$

a) along $y = x^2 + 1$

b) along line $(0,1)$ to $(0,5)$

solution

$$f(z) = (3x+y) + i(x-2y) \quad ; \quad dz = dx + i dy$$

a) $y = x^2 + 1$

$$f(z) = (3x + x^2 + 1) + i(x - 2x^2 - 2)$$

$$dz = dx + i dy$$

$$= dx + i 2x \, dx$$

$$\therefore \int_C f(z) dz = \int_0^2 \left[(3x + x^2 + 1) + i(x - 2x^2 - 2) \right] (dx + i 2x \, dx)$$

$$= \int_0^2 (3x + x^2 + 1) dx + i(x - 2x^2 - 2) dx +$$

$$i(3x + x^2 + 1) 2x \, dx - (x - 2x^2 - 2) 2x \, dx$$

$$= \int_0^2 (3x + x^2 + 1 - 2x^2 + 4x^3 + 4x) \, dx$$

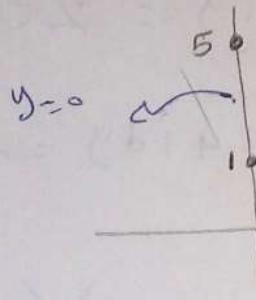
$$+ i(x - 2x^2 - 2 + 6x^2 + 2x^3) \, dx$$

$$= \int_0^2 (7x - x^2 + 1 + 4x^3) \, dx + i(3x + 4x^2 - 2 - 2x^3) \, dx$$

$$\begin{aligned}
 &= \left(\frac{7x^2}{2} - \frac{x^3}{3} + x + x^4 \right) \Big|_0^2 + i \left(\frac{3x^2}{2} + \frac{4x^3}{3} - 2x + \frac{x^4}{2} \right) \Big|_0^2 \\
 &= \frac{88}{3} + i \frac{62}{3}
 \end{aligned}$$

b)

~~$x = 0$~~ $\quad \quad \quad \mathcal{L} x = 0$



$$f(z) = y - i2y$$

$$dz = idy$$

$$\int_C f(z) dz = \int_1^5 (y - i2y) idy$$

$$= \int_1^5 iy dy + 2y dy$$

$$= i \left[\frac{y^2}{2} \right]_1^5 + y^2 \Big|_1^5$$

$$\int_1^5 f(z) dz = 24 + i12$$

* evaluate $\int \bar{z} dz$ for $x = 2 \cos t, y = \sin t$

From $t = 0$ to $t = \frac{\pi}{2}$

$$\bar{z} = x - iy = 2 \cos t - i \sin t$$

$$dz = dx + idy = -2 \sin(t) dt + i \cos(t) dt$$

$$\int_0^{\frac{\pi}{2}} (2 \cos t - i \sin t) (-2 \sin(t) dt + i \cos(t) dt)$$

$$= \int_0^{\frac{\pi}{2}} -4 \cos t \sin t dt + i 2 \sin^2 t dt + i 2 \cos^2 t dt + \underline{\sin t \cos t dt}$$

$$= \int_0^{\frac{\pi}{2}} -3 \cos t \sin t dt + i 2 dt$$

$$= \int_0^{\frac{\pi}{2}} -3 \frac{\sin 2t}{2} + 2i dt = \left. \frac{3}{4} \cos 2t + i 2t \right|_0^{\frac{\pi}{2}}$$

$$= \frac{-3}{4} + \pi i$$

تكامل على مسار مغلق

$$a) \oint_C F(z) dz = 0$$

ـ (إذ) كانت الدالة $f(x)$ ليس لها مقام ولا تحتوى على ∞
ـ (إذ) \exists x_0 هنالك مقام لا تقع داخل المنحنى $(analytic)$

$$b) \int_C \frac{f(z)}{z-a} = 2\pi i f(a)$$

$$c) \oint_C \frac{f(z)}{(z-a)^{n+1}} = \frac{2\pi i}{n!} \left. \frac{d^n f(z)}{dz^n} \right|_{z=a}$$

لذا كانت الداله تحتوي على مقام داير وصفه داخل البحarin

ادئکار

٢) تَعْوِيْنِ صَبَّا شَرَّ.

ط) فيه ذكر صدقوس كعتواص فنهم يقع داخل المحرر والباقي لا .

c) اکثر صد حوس قوت و اهمیت تقع داخل المثلث:

$$\int_C = \int_{C_1} + \int_{C_2}$$

Evaluate

$\int_C e^{10z} dz$

b) $\int_C \frac{z^2}{z-3} dz \quad |z|=1$

$\int_C \operatorname{sech}(z) dz$

$\int_C z \cdot e^{-z} dz$

e) $\int_C \frac{1}{z^2 + 2z + 1} dz$

f) $\int_C \frac{2z-1}{z^2 - z} dz$

$$|z| = \frac{1}{2}$$

1501

1) $\int_C e^{10z} dz = 0$

2) $\int_C \frac{z^2}{z-3} dz = 0 \quad |z|=1$



3) $\int_C \operatorname{sech}(z) dz = 0$

$$\operatorname{cosh} = \frac{e^z - e^{-z}}{2}$$

$$\int_C \operatorname{sech}(z) dz = \frac{2}{e^z - e^{-z}} = 0$$

8) sec 7

$$\boxed{4} \int_C z \cdot e^z dz = 0$$

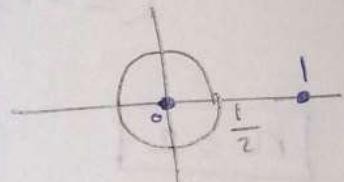
$$\boxed{5} \int \frac{1}{z^2 + 2z + 1} dz = 0$$

$$|z|=1 ; z = -1 + i$$

$$\boxed{6} \int \frac{2z-1}{z^2-z} dz = \int \frac{2z-1}{z(z-1)} dz$$

$$|z| = \frac{1}{2}$$

$$= \int \frac{\frac{2z-1}{z-1}}{z} dz \rightarrow f(z) = 2\pi i f(a)$$



$$= 2\pi i f(0) = \boxed{2\pi i}$$

$$\boxed{7} \int_C \frac{7z-6}{z^2-2z} dz ; |z| = 6$$

$$= \int \frac{7z-6}{z(z-2)} dz$$

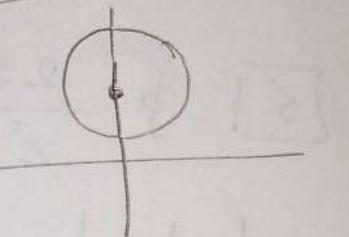
ال نقطتين داخل المحيط.

$$= \int_{C_1} \frac{\frac{7z-6}{z-2}}{z} + \int_{C_2} \frac{\frac{7z-6}{z}}{z-2}$$

$$= 2\pi i f(0) + 2\pi i f(2)$$

$$= 6\pi i + 8\pi i = 14\pi i$$

8 $\int \frac{e^{2z^2}}{(z-i)^3}$ $|z-i| = 1$



$n=2$ Cause $n+1 = 3$

~~$f = \frac{2\pi i}{2!}$~~ $f(z) = \frac{2z^2}{e}$

$$f'(z) = 4z \frac{2z^2}{e} \quad (f'(z) = 4 \frac{2z^2}{e} + 16z^2 \frac{2z^2}{e})$$

$$f''(i) = 4 \frac{-2}{e} + 16(-1) \frac{-2}{e}$$

$$= -12 \frac{-2}{e}$$

~~$f = \frac{2\pi i}{2!}$~~

$$f = \frac{2\pi i}{2!} (-12 \frac{-2}{e}) = -12 \frac{-2}{e} \pi i$$

10 $\sec r$